**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS:

= 45 = 8 are available in sum

The amount of time which takes to complete the repair on a customer's car

we can consider as X <= 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X <= 50).

Z = (X - µ)/ = (X - 45)/8.0 = **0.625**

stats.norm.cdf(z) = **0.734**

Probability that the service manager will not meet his demand will be = 100- 73.4 = **26.6% or 0.2676**.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**ANS :** Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X <= 44).

Z = (X -µ)/ = (X - 38)/6 = **1.0**

stats.norm.cdf(z) = **84.1345%**

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= **34.1345%**

Therefore, the statement that more employees at the processing center are older than 44 than between 38 and 44 is **TRUE**.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS** : Probability of employees less than age of 30 = Pr(X<30).

Z = (X -µ)**/** = (30 - 38)/6 **= -1.3333**

Thus, the question can be answered by using the normal table to find

stats.norm.cdf(z) = **9.12%**

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400 **= 36.48** (or **36** employees).

Therefore, the statement B of the question is also **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

if X ~ N(µ1, σ 1^2 ) and Y ~ N(µ2, σ 2^2 ) are two independent random variables then X + Y ~ N(µ1 + µ2, σ 1^2 + σ 2^2 ) and X - Y ~ N(µ1 - µ2, σ 1^2 + σ 2^2 )

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ~ N(aµ1 + bµ2, a^2 σ 1^2 + b^2 σ 2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ ^2) and

X1+X2 ~ N(µ + µ, σ ^2 + σ ^2 ) ~ N(2 u, 2 σ ^2 )

2X1-(X1+X2) = N( 4µ,6 σ ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- ¼ ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + ¼ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = **151.4**

Z(+0.005)\*20+100 = (-2.57)\*20+100 = **48.6**

**So, option D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Specify the 5th percentile of profit (in Rupees) for the company

Which of the two divisions has a larger probability of making a loss in a given year?

**ANS** :

We will convert the dollars into rupees

profit 1(22.5,144)

profit 2(31.5,189)

First, we determine the mean and variance for the company

μ=﻿ ﻿μ1+μ2=22.5+31.5 = **54﻿**

σ21+σ22=144+189 = **333**﻿

σ=333 ​= **18.248﻿**

second, we calculate the interval with a 95% confidence level, apply the empirical rule,add and subtract 2 deviations from the mean

(μ−2σ,μ+2σ) = (54−2∗18.248,54+2∗18.246) = **(17.504,90.496)﻿**

Third, we determine the fifth percentile

given, μ = **54**

σ = **18.248**

P(x≤x) = **0.05**

first, we computed z

z = **−1.64**

second, we can calculate x using the formula,

z = x−μ/σ third, x can be calculated as follows:

X = Zσ+μ

fourth, putting the given values, we have

x = −1,64∗18.248+54 = **24.07328 ≈ 24.07**

fifth, hence the required x is24.07​

Fourth, we determine which division is most likely to lose.

(A). profit 1

GivenWe use standardized normal distributionz=x−μ​/σ

μ1​=22.5, σ1​=12.0, x=0,

the probability can be calculated as

p(x<0)=p(x−μ/σ​<0−22.5​/12)=p(z<−1.88)=0.0301

Hence, the required the probability

p(x<0)=0.0301​

(B). profit 2

GivenWe use standardized normal distribution

z=x−μ​/σ

μ2​=31.5, σ2​=13.748, x=0

the probability can be calculated as

p(x<0)=p(x−μ/σ​<13.7480−31.5​)=p(z<−2.29)=0.011

Hence, the required the probability

p(x<0)=0.011​

(C). Profit1 is more likely to lose in a year.